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De Mazenod College – Kandana
Combined Mathematics
2nd Term Evaluation – Grade 12
Part I

Answer all the Questions   Time $2\frac{1}{2}$ Hours

01. Find the expressions for the sum and the product of roots of a quadratic equation in terms of its coefficients. If α and β are the roots of the quadratic equation $x^2 + px + 1 = 0$ then find the quadratic equation with the roots $\alpha + \lambda$ and $\beta + \lambda$ where λ is a constant.

Also if γ and δ are the roots of $x^2 + qx + 1 = 0$ then prove that

$(\alpha + \gamma)(\beta + \gamma)(\alpha - \delta)(\beta - \delta) = q^2 - p^2$

If $a > 0$ and $b^2 < 4ac$ show that for all the real values of x the expression $ax^2 + bx + c$ is positive. Then find the range of x such that $(x^2 - x - 2)(x^2 + x + 1)(x - 3)$ is positive.

02. Let $F(x) = ax^3 + bx^2 - 11x + 6$ where $a, b \in \mathbb{R}$ If $(x - 1)$ is a factor of $f(x)$ and the remainders when $f(x)$ is divided by (x-4) is -6, Find the values of a and b. Also, find the other two linear factors of $f(x)$.

Sketch the curve given by the expression $y = 1 - 2|3x + 1|$ and solve $1 + |3x + 1| = 0$

Let $g(x) = x^3 - 3abx - (a^3 + b^3)$ where a and b are real numbers. Show that (x-a-b) is a factor or $g(x)$. Find the other a factor of $g(x)$ in quadratic form. Hence, or otherwise, show that if a and b are distinct, then f(x) = 0 has only one real root Deduce that $x^3 - 9x - 12 = 0$ has only one real root and find it.

03. Using the usual notation for any triangle ABC prove that,

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

Show that, $(b^2 - c^2)cotA + (c^2 - a^2)cotB + (a^2 - b^2)cotC = 0$

$If\ a^2 + b^2 = 2b\ then, show\ that\ cotA + cotC = 2cotB$

04. Solve the following simultaneous equations,

$$sinx + cosy = 1$$
$$cos2x - cos2y = 1$$

Show that $cos3\theta = 4\cos^3\theta - 3cos\theta$ If $\theta = 18^o$ show that $cos3\theta = sin2\theta$ and $sin18^o$ is a root of $4x^2 + 2x - 1 = 0$ hence find the values of $sin18^o$ and $cos18^o$.

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Part II

Answer all the Questions Time $2\frac{1}{2}$ Hours

01. Two train stations are located 10 km away from each other. A train passing the station A with a speed of $60kmh^{-1}$ maintains its speed for 8 Km. Then it uniformly retards and comes to the rest at B. **Twelve mins** before the first train passes the station A, a second train starts its motion from the rest and accelerates at $5kmh^{-1}min^{-1}$. Then it retards and comes to the rest as the 1st train reaches the station B.

Sketch the velocity – time curves for the trains in the same diagram and show that the *second train* takes **24mins** for the journey. Also find its **maximum velocity** in kmh^{-1} and the **deceleration** in $kmh^{-1}min^{-1}$.

02. A particle is projected vertically upwards when t=0 with and initial velocity of u. The particle takes t_1 and t_2 times respectively to pass a point while travelling upwards and downwards, which is located h away from the point of projection through the trajectory of the particle.

Sketch a velocity – time curve for the motion of the particle.

Using the velocity time curve find the velocity of the particle when the time is $\frac{t_1+t_2}{2}$

Deduce that $t_1.t_2 = \frac{2h}{g}$

03. Define the dot product or the scaler product of vectors.

If $(\boldsymbol{a}+\boldsymbol{b}).(\boldsymbol{a}-\boldsymbol{b}) = 0$ does it imply that $\boldsymbol{b} = -\boldsymbol{a}$ or $\boldsymbol{b} = \boldsymbol{a}$
Justify your answer.

If the **Circumcenter** and the **Orthocenter** of a triangle ABC are given by O and H show that,

$$\overrightarrow{OH} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$$

04. If P divides the segment AB to the ratio $1:2$ and given that the position vectors of A and B are respectively $\boldsymbol{a}$ and $\boldsymbol{b}$ Find the position vector of P.

OABC is a parallelogram, here OA and OB represents the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively. L and M are the mid points of AC and CB. OL and AM intersects at X.

Show that $\overrightarrow{OX} = \frac{4}{5}(a + \frac{1}{2}\boldsymbol{b})$
CX meets OA at N. Show that $\overrightarrow{ON} = \frac{2}{3}a$

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