De Mazenod College - Kandana

10 | E | I

G. C. E. (Advanced Level) Exmination – Grade 12

March Test - 2018

26.03.2018

Combined Mathematics - I

Time 3 Hours

Answer all the questions.

(1) (i) If $(x+1)$ is factor of $f(x) = x^3 - px^2 + qx + 3$ and remainder when $f(x)$ is divided by $(x-1)$ is 16. Then find p and q.

(ii) When $a, b \in 1R$; $(x-a)^2 + (x-b)^2 = 2$ is the quadratic equation.

Whose roots are α and β. Then express $\alpha + \beta$ and $\alpha\beta$ in terms of a and b. Hence find the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(iii) Find the set of values of k ; when the roots of the equation $(x-8)(x+1) + k^2 = 0$ are real.

---

(2) (i) Find the general solution of $\sqrt{3}Cos\theta - Sin\theta = 1$. Hence find the solutions between $(0 - \pi)$

(ii) Prove that, $(Cos\alpha + Cos\beta)^2 + (Sin\alpha + Sin\beta)^2 = 4Cos^2\left(\frac{\alpha-\beta}{2}\right)$

Hence show that, $Cos\left(\frac{\pi}{12}\right) = \frac{1}{4}\left(\sqrt{6} + \sqrt{2}\right)$

(iii) Let $-\pi < \theta < \pi$ then show that $\left(Cos\frac{\theta}{2} + Sin\frac{\theta}{2}\right)^2 = 1 + Sin\theta$

Hence show that, $Cos\frac{\pi}{12} + Sin\frac{\pi}{12} = \sqrt{\frac{3}{2}}$

And $Cos\frac{\pi}{12} - Sin\frac{\pi}{12} = \frac{1}{\sqrt{2}}$

Then show that, $Sin\frac{\pi}{12} = \frac{\sqrt{13}-1}{2\sqrt{2}}$

(3) (i) Find the general solutions of $2Cosx - Sin2x + Sinx - 1 = 0$

(ii) State the Sin rule and show that, $(b+c)\ .\ Cos\left(\frac{B+C}{2}\right) = a\ Cos\left(\frac{B-C}{2}\right)$

If $b + c = \mu a$ the prove that,

$$Cot\frac{B}{2}\ .\ Cot\frac{C}{2} = \frac{\mu+1}{\mu-1}$$

(iii) Draw the rough sketch of the function $y = Cosx + Sinx$ for $-\frac{5\pi}{4} \le x \le \frac{3\pi}{4}$

Hence deduce that $x = \frac{\pi}{4}$ is the only solution to the equation

$$Cosx + Sinx = \frac{4\sqrt{2}x}{\pi}$$

---

(4) (a) Solve $Tan^{-1}\left(\frac{x+2}{x+1}\right) - Tan^{-1}\left(\frac{x-1}{x+5}\right) = \frac{\pi}{4}$

(b) Main values for the inverse functions find the value of $Cos\left[Tan^{-1}\left(-\frac{3}{4}\right) + Sin^{-1}\frac{5}{13}\right]$

(c) Find the partial fraction of $\frac{2x^3 - 3x^2 - x + 5}{2x^2 + x - 1}$

---

(5) (a) Prove that $\log_a b = \frac{1}{\log_b a}$

Hence solve the equation

(i) $\log_2 x - 2\log_x 2 = 1$

(ii) Prove that, $\frac{1}{1+\log_b a+\log_b c} + \frac{1}{1+\log_c a+\log_c b} + \frac{1}{1+\log_a b+\log_a c} = 1$

(b) Solve $2^{2x} + 7.2^x - 8 = 0$

(c) Prove $\ln\left(\frac{a}{b}\right) + \ln\left(\frac{b}{c}\right) + \ln\left(\frac{c}{a}\right) = 0$

---

(6) Prove that if polynomial $f(x)$ is divided by $(x - a)$ then remainder is $f(a)$. When the polynomial $f(x)$ is divided by $(x-\alpha)(x-\beta)(x-\gamma)$, where α, β and γ are unequal real numbers the reminder takes the form $A(x-\beta)(x-\gamma) + B(x-\alpha)(x-\gamma) + C(x-\alpha)(x-\beta)$ Express the constants A, B and C in terms of α, β, γ $f(\alpha)$, $f(\beta)$ and $f(\gamma)$.

Hence find the value of the constant k for which the reminder when $x^5 - kx$ is divided by $(x+1)(x-1)^{(\ )}(x-2)$ contains no term in x.

- Sampath Dikkumbura -

De Mazenod College - Kandana

10 | E | II

G. C. E. (Advanced Level) Exmination – Grade 12

March Test - 2018

26.03.2018

Combined Mathematics - II

Time 3 Hours

Answer all the questions.

(1) (a) Define the dot product for the vectors $\underline{a}$ and $\underline{b}$.

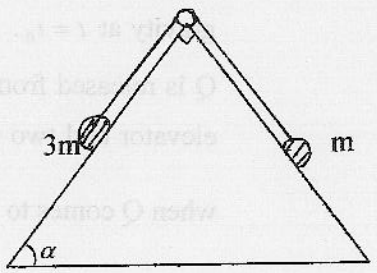
Let ABCD is a parallelogram and $\overrightarrow{AC} = a$ $\overrightarrow{BD} = \underline{b}$, then show that,

$$B\hat{A}D = Cos^{-1}\left[\frac{|a|^2 - |b|^2}{|\underline{a}-\underline{b}| * |\underline{a}+\underline{b}|}\right]$$ by considering the dot product.

(b) Let the position rectors of the points A and B are $\underline{a}$ and $\underline{b}$ with respect to a fixed point O. Show that the position vector of any point on AB is in the form of $(1-\lambda)\,\underline{a}+\lambda\underline{b}$; where λ is a scalier. Hence find the position vector of point c; where $AC : CB = 2 : 3$

---

(2) A smooth wedge is fixed on a horizontal surface as shown in the figure. Two particles mass 3m and m connected with a string and passing over a smooth pulley are kept on the wedge and released from rest. Find;



(a) Common acceleration.

(b) Tension of the string

(c) Resultant force acting on pulley.

---

(3) A lift starts from rest and move uniform acceleration a up to its maximum velocity u ; then travels with uniform velocity and comes to the rest with uniform deceleration $3a$. If the total distance traveled by lift is h in total time t. Show that the time traveled in maximum velocity is $\sqrt{t^2 - \frac{8h}{3a}}$.

(4) (a) In a oxy plane two forces act on the points (1 , 2) and (-2 , 1) are $5i+7j$ and $-2i-4j$ respectively. Find the magnitude direction and the line of action of the two forces.

(b) ABCDEF is a regular hexagon of centre O and length of a side a meters. Five forces P, 2P, 3P, 4P and 5P Newtons act along the sides AB, BC, CD, DE, EF respectively in the directions indicated by the order of the letters. Three new forces Q, R, S Newtons acting along the sides AF, FO, OA respectively, of the triangle AFO are added to the system. Find the values of Q, R and S in terms P in order to the combined system is,

(i) in equilibrium

(ii) equillant to a couple of moment Pa $\sqrt{3}$ Nm in the same sense ABC.

---

(5) An elevator starts its motion from rest at time $t = 0$ and moves vertically upwards with uniform acceleration a. A man who is in the elevator releases a particle p from rest under gravity at $t = t_0$. AT the instant when particle reaches its maximum height, a second particle Q is released from rest under gravity. Sketch the velocity time graphs for the motions of the elevator and two particles P and Q on the same diagramme. Hence, show that at the instant when Q comes to instantaneous rest, the velocity of p is $ato\left(\frac{a}{g}+1\right)$

---

(6) A vehicle of mass Mkg pulls a trailer of the same mass by light inextensible cable along a straight horizontal road. The resistance to the motion of the vehicle and motion of the trailer are R and $2R$ Newtons respectively. Show that the instant when the engine of the vehicle is working at power P Kw and the vehicle is moving with speed Vms^{-1}, the tension of the cable is $\frac{1}{2}\left(R+\frac{1000\,P}{V}\right)$ Newtons.

- Sampath Dikkumbura -