30.07.2019

De Mazenod College - Kandana

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G. C. E. (Advanced Level) Examiition

Grade 12

Jyly Test - 2019

Combined Mathematics I

Time
2 ½ hours

Part A

Answer all the questions in this paper itself.

01. If $2x^2 - 5x + 14 \equiv a\ (x-1)\ (x-2) + b\ (x-1) + c$ find the constants $a,\ b,$ and c.

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02. When $p(x) = (\lambda - 2)x^2 - 3\ (\lambda + 2)\ x + 6\lambda\}$ wher $\lambda \in R$. If $P\ (x)$ is positive for all values of x. Find the least integer value for λ.

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03. $\lim_{x \to 0} \dfrac{(x+2)\sin^2 2x}{\sqrt{5+Kx^2}-\sqrt{5}} = \sqrt{5}$ then find the constant of K.

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04. If $y = \tan^{-1}(\sec x + \tan x)$ then show that $\dfrac{2dy}{dx} - 1 = 0$ also find y and $\dfrac{dy}{dx}$ when $x = 0$.

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05. $x = \cos t$, $y = \cos Kt$, $K \in R$ Find $\frac{dy}{dx}$ and $\frac{d^2 y}{dx^2}$, also show that $\frac{dy}{dx} = -4$ when $t = \frac{\pi}{3}$ and $K = 2$

06. Solve $\frac{x + 3}{x - 1} \geq x$

07. If A is the intersection point of two straight lines $2x+y-1=0$ and $2x+3y+4=0$. Without finding the coordinates of A. Show that the straight line passing through point A and parallel to $2x+3y-1=0$ is $10x+15y+13=0$

08. A curve is defined by the equations $x=t^2$ and $y=at^3-t^2$ where $a \in R^+$, If the tangents drawn to the curve at the points given by $t=1$ and $t=-1$ are perpendicular to each other. Find the value of a.

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09. If $\alpha + \beta = \frac{\pi}{4}$; then show that $(1 + \tan\alpha)(1 + \tan\beta) = 2$

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10. $ABCD$ is a trapezium with AB and CD are parallel where $D\hat{C}B = 90^0$. If $A\hat{D}B = \theta$ $BC = p$ and $CD = q$, show that $AB = \dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$.

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- Sampath Dikkumbura -

30.07.2019

De Mazenod College - Kandana

10	E	I

G. C. E. (Advanced Level) Examiftion

July Test - 2019

Grade ~~13~~ 12

Combined Mathematics I

Part B

Answer four questions only.

(11) (a) Show that $x^2 + (3K - 2)x + K(K-1) = 0$; Where $k \in \Re$ he equation has two distinct real roots.

i. Find the values of K when the different of two roots ~~eg. Is Z~~ are Z.

ii. If α and β are the roots of above equation when $K = 3$; then show that quadrate equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is $6x^2 + 7x + 1 = 0$

(b) Prove that $\log_a b = \frac{\log_c b}{\log_c a}$ and reduce formula $\log_a b = \frac{1}{\log_b a}$

i. Solve $\log_3 x + \log_x 9 = 3$

ii. Show that, $\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz} = 2$

(12) (a) If a and b are distinct real numbers. Find A and B for following

$$\frac{1}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

By using suitable for above x, a and b, find the partial fractions of

$$\frac{1}{(x^2 + a^2)(x^2 + b^2)}.$$

(b) Remainders when $p(x)$ is divided by $(x-1)$ and $(x-2)$ are 5 and 7 respectively. Find the remainder when $p(x)$ divided by $(x-1)(x-2)$.

If $p(x)$ is a expression of degree 3 and the coefficient of x^3 is 1. Also $p(x)=0$ has a root of -1 then find $p(x)$.

(c) Sketch the graph of $y = 1 + |x-1|$ and $y = \left|\frac{x}{2}+1\right|$ in a same diagramme. Hence solve the inequality $\left|\frac{x}{2}+1\right| - |x-1| > 1$.

(13) (a) If $y = \sin 5x.e^{4x}$ then show that $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 41y = 0$

Hence show that $\frac{d^2y}{dx^2} = 40$; When $x = 0$

(b) If $f(x) = \frac{x^2+4}{x^2-4}$ then show that, $f^1(x) = \frac{-16x}{(x-2)^2(x+2)^2}$

Sketch the graph of $f(x)$ by indication the asymptotes and the turning points. Also find the range of K when $f(x) = K$ has two distinct roots.

(c) Rate of increase the radius of a sphere is $\frac{1}{3}\,cms^{-1}$. Find the rate of increase the volume when radius is 6 cm.

(14) (a) If the sides of a parallel gram are having equations of $x-y-2=0$, $x-4y-4=0$, $x-y+1+0$ and $x-4y+3=0$. Find the equations of two diagonals, without finding the coordinates of vertices

(b) Find the acute angle bisector of $2x+y+1=0$ and $x+2y+1=0$. Find the position of two points $(2,-1)$ and $(3,2)$ with respect to the above angle bisector.

(15) (a) Show that $\sin^6 x + \cos^6 x = 1 - \frac{3}{4}\sin^2 2x$

Hence solve the equation $\sin^6 x + \cos^6 x = \frac{7}{16}$ in the range of $-\frac{\pi}{6} \le x \le \frac{\pi}{2}$.

(b) i. Show that $2\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

ii. $2\tan^{-1}\left(\frac{5}{12}\right) = \tan^{-1}\left(\frac{120}{119}\right)$

Also show that ~~Hence reduce~~ $\tan^{-1}\left(\frac{120}{119}\right) - \frac{\pi}{4} = \tan^{-1}\left(\frac{1}{239}\right)$

(c) State the "sin rule" for triangle *ABC*.

Hence show that,

$(a+b)\,.\,\sin\frac{c}{2} = c\,.\,\cos\left(\frac{A-B}{2}\right)$

- *Sampath Dikkumbura* -

30-07-2019

De Mazenod College - Kandana

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G. C. E. (Advanced-Level) Examiition **Grade 12**

July Test - 2019

Combined Mathematics II

Time
2 ½ hours

Part A

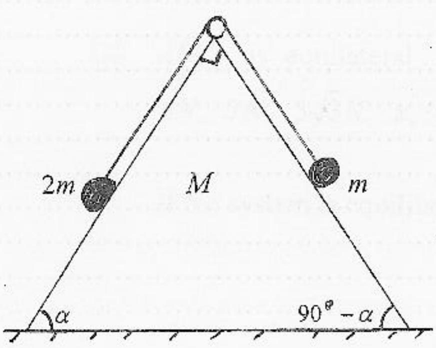
Answer all the questions.

01. A particle A is projected by velocity u an angle $\hat{\theta}$ to the horizontal at the same time another particle B is projected from a point which is height h from the floor by velocity v to the to the horizontal. If two particles are collided each other. Show that $v = u\cos\theta$ and $t = \frac{h}{u}\cos\theta$; where t is the time taken to the collision. Also find the vertical distance to the point of collision from the floor.

02. A lorry with mass $10^6\,kg$ has the power $35\times10^4\,kw$. If the resistance force on lorry is $25Nkg^{-1}$. Show that the acceleration of the lorry, when it moves by constant velocity $36kmh^{-1}$ along upward direction of an inclined rod at an angle 100 : 1 is 9.9 ms^{-2}.

03. A and B are two particles with mass $2m$ and $3m$ move to the opposite direction with velocity $7u$ and $3u$ respectively. Show that impulse of the collision is $12\,mu\,(1+e)$ where e is the coefficient of restitution. If the particle A comes to the rest after the collision find the value e.

04.

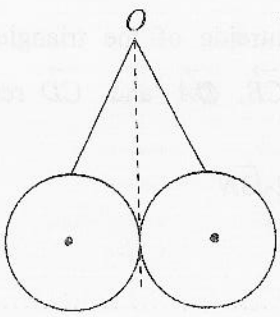


A wedge pf mass M is fixed to a horizontal table. Mass $2m$ And m are connected by a string and passing over a smooth pulley are kept on the wedge as shown in the figure. Show That that the common acceleration of the particle when the system is released from rest is $\frac{g}{3}(2\sin\alpha - \cos\alpha)$ and ~~$a = \frac{g}{3}(2\sin\alpha - \cos\alpha)$~~ and $T = \frac{2mg}{3}(\sin\alpha + \cos\alpha)$ where T is the tension of the string.

Define the dot product of two vectors $\underline{a}$ and $\underline{b}$. Let $ABCD$ is a parallelogram where $\overrightarrow{AC} = \underline{a}$, and $\overrightarrow{BD} = \underline{b}$ by considering dot product of two vectors.

Show that, $B\hat{A}D = \cos^{-1}\left\{\dfrac{|\underline{a}|^2 - |\underline{b}|^2}{|\underline{a}-\underline{b}|\,|\underline{a}+\underline{b}|}\right\}$. If AB and AD are pendicular reduce that

$|\underline{a}| = |\underline{b}|$



Two spheres of weight w and radius a are connoted by two strings of lengths $2a$ is hang on a point O as shown in the figure.

i. Mark the forces act on the system.

ii. Show that, $T = \dfrac{\sqrt{2}w}{4}$ and $R = \dfrac{\sqrt{2}w}{4}$ by use of a triangle of forces where T is the tension of strings and R is the reaction on the spheres.

07. Define the angle of friction. A rod is weight w and length $2a$ is kept on rough horizontal floor and the other end is touched with the same roughness of vertical wall. If the angle of the rod with vertical is $\hat{\theta}$. Show that $\theta = 2\lambda$ where $\hat{\lambda}$ is the angle of friction. Show the normal reaction on the wall is $\frac{w}{2}\tan\theta$.

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O8 ABC is equilateral triangle with side 2m' O is the contreide of the triangle. Forces $8N$, $7N$, $3\sqrt{3}N$. x, y and Z act along $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{BO}$, $\overrightarrow{OB}$, $\overrightarrow{CB}$, $\overrightarrow{OA}$ and $\overrightarrow{CD}$ respectively.

If the system is equilibrium. Show that $x = 1N$, $y = 5\sqrt{3}N$, $Z = 2\sqrt{3}N$

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C. Maths -

09. If position rectors of A and B with respect to a point O are $\underline{a}$ and $\underline{b}$. Show that position vector of any point on AB can be written in the form of $(1-\lambda)\,\underline{a}+\lambda\underline{b}$. When λ is a scalar. Hence find the position vector of point C such that $AC : CB = 2:3$. Also show that the position vector of mid point AB is $\frac{1}{2}(\underline{a}+\underline{b})$

10. Probabilities of shooting to a target for two children are $\frac{1}{3}$ and $\frac{1}{4}$ .. If one opportunity is given to them each .

i. Show that the probability of shooting to the target exactly one of them is $\frac{5}{12}$.

ii. Find the probability of shooting to the target by the first child, it is given that exactly one child is shot to the target.

- Sampath Dikkumbura -

30.07.2019

De Mazenod College - Kandana

10	E	II

G. C. E. (Advanced Level) Examiftion **Grade 13**

July Test - 2019

Combined Mathematics II

Part B

Answer four questions only.

(11) (a) Train starts from rest from station A and finish at B. It travels distance d^1 by constant acceleration, afterward maximum velocity and finally distance $\frac{d^1}{2}$ by constant deceleration then rest at B. If the distance between A and B is d, the maximum velocity reached by the train is v and the average velocity for whole journey is u , then sketch the velocity time graph for the motion of the train. Hence show that $\frac{u}{v} = \left(\frac{2d}{2d + 3d^1}\right)$ Also to occur the above motion show that, $d > \frac{3d^1}{2}$.

(b) An elastic ball is released from a point which is height h from the floor. If the coefficient of restitution between floor and the ball is e. Find the velocity of the ball after three collision with floor. Find the total time for the above motion.

(12) (a) A bullet is fixed from a point P on the floor by velocity v at angle 45° to the horizontal. If the horizontal and vertical distant from point P is x and y. Then show that $y = x - \frac{gx^2}{v^2}$. If the bullet hits the floor at point Q which is horizontal desistance $x = a$.

Another bullet fired from point P by velocity u & anangle 45^0 with the horizontal passing through a point which is vertically height h from point Q. Then show that, $u^2 = \frac{v^4}{(v^2 - gh)}$

(b) A lorry with mss $M\,Kg$ moves along a horizontal road by it's maximum velocity Ums^{-1} and it's power of the engine is $H\,Kw$. When the lorry ascends along an inclined road at angle α to the horizontal. It's maximum velocity is Vms^{-1}.

Show that $H = \dfrac{UVMg \sin \alpha}{1000\,(U - V)}$ assume that the resistance against to the motion is always constant.

13. (a) Coplanar three forces $2p(\underline{i}+3\underline{j})$, $-3p(-\underline{i}+4\underline{j})$, $-2p(\underline{i}-\underline{j})$ act on the points $A(1, 2)$, $B(-1, 3)$ and $C(4, -1)$ in $o\,x\,y$ Cartesian plane. Represent the above system of forces by it's components. Hence show that the resultant force of the systemis is $5p$ and find the direction and the line of action of the resultant force. Find the coordinates of the intersection point of x is and the resultant force. If additional force of $5p$ is added to the system at $(0, 0)$, then the system is reduced to a couple. Find the moment of the couple and the sense of it.

(b) If position vectors of points A and B are with respect to a point O are $\underline{a}$ and $\underline{b}$. C is a point such that $\overline{OC} = \underline{a} + 2\underline{b}$ and D is the mid point of BC. Then show that $2\overline{OD} = \underline{a} + 3\underline{b}$. If E is the intersection point of OD and AB. Then show that $AE : EB = 3 : 1$

14. (a) CB is a light rod with length $2a$ end C is fixed to a point on a horizontal table. Another rod BA length $2a$ and weight w is fixed to the point B and A is kept on the table in order to make angle $\hat{\theta}$ with AB and the vertical. If the system is in equilibrium show that $\tan\theta \leq 3\mu$; when μ is the coefficient of friction with t the point A.

(b) None uniform rod AB with weight w and length $\sqrt{3}a$ completely kept inside of a fixed sphere of radius a by making an angle 60^0 to the vertical. If the system is equilibrium. Show the reactions at the ends of rods are $\dfrac{2w}{\sqrt{3}}$ and $\dfrac{w}{\sqrt{3}}$.

15. (a) A box x has 2 red balls and 4 white balls. Another box y has 2 red balls and 2 white balls. A ball is drawn from x at random and put in to y. From y 2 balls are drown successively without replacement. Construct a tree diagram and find the probability that,

i. All three balls drown are red

ii. At least one white ball is drawn

iii. Drawn three balls are red it is given that drown three balls are same colour.

(b) A certain illness x has only one of the two symptoms A and B. It is known that in the usual notation $P\left(\frac{x}{A}\right) = 0.2$ and $P\left(\frac{x}{B}\right) = 0.8$

In a certain population 40% have symptom A and the remaining 60% have symptom B. Calculate the probability that a randomly picked a person has illness x. Also show that the probability of symptom B being shown, given that the patient is suffering from illness x is equal to $\frac{6}{7}$.

- Sampath Dikkumbura -